MAY 5, 1969 ROBERT H. ALLEN

Mathematicians delight in the intricate web of relations that can be found among numbers. Here is an entrancing game with a seemingly infinite variety of plays within the framework of simple and unalterable ground rules.

Just imagine what must have led from the primary and it must have been nearly self-evident concept of quantification, what train of exploration must have led up to the proposition that the sum of squares of the sides of a right triangle is equal to the square of the hypotenuse. Can there be anything more beautiful in poetry or music than that single thought?

Think of this: if you prepare chart paper on a logorithmic scale, then the slope of the line representing proportionate change will be the same whether the range of numbers involved be x or an order of magnitude greater. If ten incidents change by one, that is a ten percent change. If 100 incidents change by one, that is a one percent change. On ordinary graph paper - laid out in equal squares chosen to represent one unit each, each of these changes would produce the same slope. On a piece of logorithmic graph paper, where the squares vary in size, the graph would show the viewer that the rate of growth of one to one hundred was much less than the rate of growth of one to ten. On the other hand a logorithmic graph would clearly disclose the opposite situation where a constant rate of growth prevailed and the size of the base grew tenfold. A non-logarithmic chart would give the impression of accelerating growth in this case. Both pictures are true. The rate of growth relative to the current total is the same but the absolute growth is increasing to sustain the rate. The chart selected will depend on the user's purpose. To get a useful answer, he must understand what question he needs answered.

It is not my purpose tonight to review the progress of mathematics. Suffice it to say that it did not spring full grown from the head of Zeus and, that for such an ancient art, it still

continues to evolve amazingly flexibly and fruitfully.

The fascination of mathematcis lies in its remarkable and beautiful order and students of the subject have pursued it for its own sake. But from time to time nature has appeared to be a subtle mathematician. For example, the planets describe ellipses about the sun, the ellipse being an interesting figure to construct which was defined long before solar orbits were thought of. The orbits being described, it then appeared that these ellipses (in which an equal time of travel sweeps out an equal acreage of space on the plane) do indeed coincide with what gravitational theory requires. But it was the ellipse and its characteristics (purely of mathematical interest) and then the fact of elliptical orbits described that led to the enunciation of the gravity princiapl, not the reverse.

As mathematics looks about, it tries to find ways of describing events that can be reduced to the simplest and most orderly statements possible. It is remarkable what order can be found in an apparently complete dissaray. Mathematicians even construct sets of random numbers which serve, among other things, to test whether some phenomenon is in fact random in its incidence, and to explore the characteristics of such phenomena.

The evolutionary process is one in which random happenings play a predominate role. For a time it seemed strange that this could be so and a good deal of careful thought was required to solve the puzzle.

Students of evolution wanted to understand the effective mechanisms. This involved at first a great amount of speculation without the benefit of the detailed understanding afforded by relatively recent investigative tools. Early in the game it was a question of drawing inferences from phenomena observable on a very gross scale.

By now it seems reasonable to view the evolutionary process as being guided by three

factors or forces. First is the past history of the process itself. Changes that occur are limited by what exists to change and by what capabilities for change are present within existing organisms. Second is chance. Changes in existing organisms result from mutations and from the regular chance construction of nrew personalities involved in the reproduction process. The third factor is adaptation. By this is meant that changes that confer an advantage - given the existing environment - tend to spread through the population. Conversely changes that are disadvantageous, at the moment, do not become widely propagated.

Playing the background of all this must be recognized variations in the environment which may, from time to time, alter the nature of changes that are advantageous to different organisms.

These generalizations in turn led to a closer understanding of the mechanisms for change. Today one may open up a magazine of general circulation and read about - or even see pictures of the latest discoveries in detailing the remarkable devices by which the structure of complex life forms is encoded and reproduced from generation to generation.

Chance is a very interesting thing and much misunderstood. For example the chances are 635, 013, 599, 599 to one against your being dealt a hand of 13 spades the next time you sit down at the bridge table. The chances, however are exactly the same against your being dealt any listed thirteen cards no matter how mediocre or routine a hand you describe. The fantastic odds against your getting thirteen spades is merely a function of the number of possible hands that there are in a deck. What makes the high odds is looking for a specific predetermined combination to occur at the next deal.

In bridge a hand of thirteen spades happens to be one that would give the recipient a good score and make him into a n obnoxious bore for the rest of his life. But no one supposes that the purpose of bridge hands being dealt is to await

this event. One argument against contemporary theories of evolution is that the odds against the present deployment of animal life are too high. The odds against any specified deployment would be just as bad of course and the argument rests for its sense of conviction at least in part on the misuse of mathematical probabilities.

In this audience it would be a better than good even money bet that there are two men whose birthdays fall on the same date. This seems strange at first, but think a minute. *f there were two men the odds would be 365 to one against one's birthday falling on the same date as the other's. Each time another man joins the group the odds go down, a third man having 2 chances in 365 for a match with one of the others, or only about 183 to one against. By the time about twenty men are in the room, each has about one chance in twenty of finding a match. And with a universe of twenty it can't be a bad bet that one of these twenty-to-one shots will be a winner.

The purpose of these examples is to lead us to the supposition that some sense of the workings of the rules of probability and some understanding of the conditions of their application would be useful in helping us to evaluate a number of situations.

Here is an interesting thing. A man named Heisenberg, in 1927, demonstrated the "Principle of Uncertainty". What it amounts to is a proof that we could not observe certain things about an individual electron. This tiny bit of matter flits about at the speed of light and a light sufficiently strong to show it would displace it. Electron behavior can be quite accurately described when we are considering large groups of them. The fewer involved the more indeterminate things become and a single one is quite evanescent. So we are at the point where ultimate reality in small matter can only be usefully seen in terms of probability. Electron behavior is something like the drunkard's walk, a term used to describe a set of artificially constructed movements of random distance and direction.

At the other end of the size scale, looking at the great mass and distances of the universe, again we run into mathematics. Reality eludes metaphor. It completely escapes the limits of human sensory depiction and can only be usefully described in formulae from the mathematician's art. Further it becomes clear that the elucidation of a standpoint is a necessary element in the depiction. There is no such thing as an observation without an observer. What we observe are things in relation to ourselves. So knowledge seems to be both limited and relative in its basic character.

Without pushing the parallel too far, it is none-the-less interesting to note that in some respects human behavior can be looked at in the same way. If we wonder what an individual will do in a- certain circumstance we may find it difficult to determine. But given a large number of individuals it is possible on occasion to determine very accurately what they will do, that is to determine very accurately what proportions of them will follow possible courses of action.

An amazing number of situations lend themselves to mathematical analysis and can be explained - perhaps described is a better word - more effectively, more usefully in mathematical terms than in any other. We are sometimes led to wonder whether the mathematical description is not the reality and the pragmatic search for rational causes merely an exercise.

There are two pitfalls, however, in rushing into a too delirious embrace of this proposition. First is the matter of data and the second is the matter of application. I once saw a curve drawn that seemed to describe the incidents of some events rather well. But my mathematician friend was not satisfied with some irregularities and went back to his study, emerging later to show that what seemed at first examination to be a single describable curve with a few dots out of place was in reality at least two curves and perhaps more. The first and more simple apparent result was merely an average that could not safely be relied upon for predictive description. In fact in that situation

we finally discovered that we didn't seem to have adequate data to find a set of curves that would usefully picture the variable factors at work.

Today it appears that one must at least question whether the exponents of the new economics have sufficient data to make econometrics useful in the fine tuning of the national economy. As we are well into the third quarter since the measures taken last summer were supposed to slow things down - and things still roll along all too merrily, we can be forgiven for suggesting that perhaps the data at hand is not sufficiently refined.

The other pitfall I have suggested we must avoid is in the matter of application. Some of what has gone before is pertinent here. The finalist argument regarding evolution is a mis-application of probability theory. Let me elaborate a hypothetical example. A man walks down a road, never turning back, but coming from time to time upon choices of paths at forks in the road. From the man's perspective the choices he keeps running upon are quite small so his election of them is not deeply considered but is random.

We can say several things about this example. First, looking at a map of it we know that certain destinations are no longer possible at all after some of the possible turns have been passed. We would have no need of or use for probability calculations with regard to these.

Second, we might see from the scale of the map and the speed of the man that he will not get to any place by nightfall. Darkness will see him still traversing the road, randomly taking this or that turn, and surely he will simply have to sleep there en route under the stars because for this day at least the road leads nowhere.

Third we might realize that, looking at just this single traveller, we have no idea at all what next turn he may take, or what next after that and we can hardly guess at all where he will be an hour from now.

But, fourthly, if there are a thousand men on the road we can, calculating from the rules of the game, tell quite accurately how many will be at each of four or five points in the maze where night might overtake them.

This illustration of paths or roads with forks being constantly encountered seems useful to me and I shall return to it. But lets think for a moment of some things that mathematics has done to our thoughts. The views of the world - illuminated by mathematically derived theories and buttressed by experiment and observation, that science has produced, have succeeded in overwhelming all prior speculation and philosophy.

Not so long ago arguments between materialists and idealists could be fought on more or less even terms. At least each side could drive dangerous holes into the specific formulations of the opposition. Mechanism as a philosophy might be convincing except for one problem: Newtonian physics provided no credible process. The proponents of mechanism could not account for the mind. Mechanidal causality as a proposition could not account for the apparent ability of nothing to have an effect on something. Yet the argument that mind was only a product, having no role in the causal chain, was so repugnant to feeling, observation and common sense as to be useless.

Determinism provided an unacceptable world view and yet its premises seemed logically unassailable. The sorts of causes available to consider could not be accepted as exclusive and yet they seemed indeed so to be. At the same time the idea of effects without causes was an equally unacceptable alternative.

It is no wonder in these circumstances that the study of philosophy tended to become a study of the history of philosophy. Contemporary speculation had run upon the impasse of a group of paradoxes and creative philosophy turned away in frustration from the fundamental questions of existence. The time approaches when some new Santayana may try a synthesis again. This great

man's The Life of Reason first appeared in 1905 and even the second edition in the early twenties could not have been prepared with the developments since Einstein in view.

It should be admitted that there is a problem, or a question that can be stated as a problem, that no amount of discovery can dispose of. This is the idea of a first cause. Here the human mind can find deep fascination. Take off backwards any way you choose and at the end of speculation or reconstruction still lies the question: and before that?

The principal use of this gambit has been as a theological argument. The Divine Being is, by definition, endowed with qualities that preclude - or seek to preclude questions as to His antecendents and the unbeliever is thus confounded.

Contemporary skeptics are inclined to attempt to dispose of first cause by declaring it to be irrellevant. It is suggested that a question whose answer has no consequences is not really a question at all. But this is not fully satisfying. Possibly some day it will be. Today we do respond to the question and it has at least the consequence that many people consider it relevant to their world view. We are not ready to dismiss it as being equivalent to the search for a positive whole number smaller than "one".

So if you will forgive me I shall proceed leaving this small question unanswered and try to see what we may conclude even so.

The idea of chance, expanded into the great mathematical branch that deals with probabilities is central to an understanding - to a useful and even a possible understanding of what makes up our world. It starts with the movement of electrons, is a principal determinant in the evolution of living forms and is most useful in describing a multitude of phenomena including the general results of actions that individuals see as stemming from carefully reasoned decisions. I have not been exposed yet to any study working backwards and con-

sidering the implications of this in the analysis of individual motivation, except for the work of Mr. Samuel Lubell on voting preferences. He sees much of the decision resting on essentially irrational grounds.

Santayana pointed out interestingly in this regard that reason requires an irrational base of material on which to operate and statisticians recognize that their craft is most useful in the understanding of say general movements comprising myriads of individual motions that are, within constraints, random.

Sophisticated mathematical techniques more and more prove useful in illuminating the operation of events. And as mathematicians have attempted to apply their craft to phenomena new mathematical forms have developed, first perhaps as rules of thumb that have served a purpose and then as general areas of the discipline.

Numbers can be useful even when mysterious. Every school boy learns to use 3.1416 and to call it "pie" but few are concerned with the mystery of why this absurd number really produces the area of a circle when multiplied by the square of its radius.

Similarly consider Plank's constant, the result of dividing the amount of emitted energy in any process of radiation by the frequency. It is a small number, a decimal point followed by twenty six zeros and roughly 6624, and it is a universal constant. No explanation has been given, but the mathematics involved are very useful and make it possible to get a handle on (I can hardly say understand) a lot about the nature of matter and energy.

There are some lessons to be learned. One certainly should be to advise young people to study math. For it is growingly true that comprehension of the world lies more and more in numbers and less and less in the colorful collection of subjective phantasies with which we are wont to clothe reality.

The detailed knowledge of the world that has been accumulating has effectively spoiled some of the most enjoyable speculation of generations of philosophers. Careful distinctions between mind and body retreat before growing evidence that there is no distinction. Realism versus idealism is not a rational subject for discussion anymore. And even mechanism versus free will is a hazardous area for unbridled speculation.

The limits of knowledge today are more than ever interior limits. The tools of analysis and exploration are magnificent. The willingness to frame questions lags. Par too often the search for understanding is impeded not by the difficulty of the subject but by the bias of the student. It is amazing to me that a major school of thought on the subject of human character is that there is no such thing. That is, an entirely reputable group of students holds that man is born into the world with no predispositions or instincts, or call them what you will, but is entirely a product of his environment. The fact that such a view of our nature is to be found in Marx - and apparently led to the Lysenko fallacy - may have something to do with the violent reaction to any contrary view. Certainly Mr. Robert Ardery's speculative popularization of the work of Lorenz and others has been attacked with such virulence and such absolute rejection as to remind one of the Baptists reaction to Darwin.

Not alone, but prominent among reformers, Marx recognized that a rigid view of human nature, the acceptance of a rigid list of characteristics, would be fatal to any proposal for major reform. If the classic Christian view of man as a sinner, or some other set version of man were true, then it would follow that man's relationships, his polity and economy, were natrual outcomes of his nature and not subject to revision, certainly at least not subject to radical restructuring based on rational principles.

This concern does not, to my mind, lead necessarily to the proposition that the entire content of man's character is the result of education, that there is literally no inherent set of pre-

dispositions at all. Nor do I agree that it follows from the great variety of ways in which humans have organized their communities, that there is no such thing as natural or unnatural situations. The fact is that we are just on the threshhold of coming to grips with the infrastructure of human conduct. We are in the position of seeing a growing body of bits and pieces of knowledge, but of continuing to have to base our generalizations on very crude observations.

Further data on this question will be forthcoming. Certain types of behavior can be altered by drugs. The receptivity of infants to imprinting at certain ages is being studied. We are learning much more about the mechanics of the genetic process. Some interesting studies have been made of the relationship of an extra chromosone of a type to height and the commission of certain crimes of violence.

Let me return a moment to my example of the man and the forks in the road. Here I arbitrarily supposed that the decisions he would make would not follow from any particular pattern of thought but would be, for our purposes, random. It is quite obvious that in that example a particular man might consistently take the left turn at each fork in the road or make some other systematic selection of turns. Another case may serve to give more verisimilitude to the idea of the effective randomness of many decisions.

Watch a lady in a supermarket examining bags of oranges. She will look at several, lift a couple and finally put one in her cart. Watch the pile until it is down to only a few and you will see a large group of ladies going through the process of making a decision. In the end though the oranges have all been sold and the only material questions concern an occasional bag that has a soft orange in it or such things as whether the produce manager guessed correctly the number of cases of oranges he would need that day.

We spend a lot of time in decisions that are barely material to ourselves and which viewed

from any broader perspective make no difference at all. At the same time important matters for us, such as the choice of a wife or a job or the number of children we have are determined as much by a series of random events as by any conscious consideration on our part.

The role of logic or reason or even of irrational choice in the world looks different when we try to examine the individual case - a sort of single electron - than it does when we look at large numbers of choices made by large numbers of individuals. It is probability theory that reconciles the idea of free will and the idea of determinism. This concept permits us to understand how the world can be so different when seen from the two points of view and how it can be that both views are correct.

From this new perspective it may be possible to reconstruct philosophy. For one thing we have a powerful tool to examine - if we will - questions of the nature ofhuman character with less prejudgment and more opportunity to test our conclusions.

Santayana felt sixty years ago that what was known about what goes on in men's minds was "at best poetry". "If," he says, "we attributed to other men only such obvious reasoning, sound judgement, just preferences, honest passions, and blameless errors as we discover in ourselves, we should take but an insipid and impractical view of mankind." He goes on to point out that "in fact, we do far better," and end up with "a notion of our friends' natures which is extremely vivid and seldom extremely flattering."

When we combine more detailed knowledge of ourselves and a better understanding of the mathematics of our interactions then perhaps we are on the way to using The Drunkard's Walk to illuminate the dark corners of the mind and to finally move toward a view of the world that will be something we can live with in harmony and joy.

The attainment of this harmony is not

merely a pleasant hope. For in the end our ability to speak with each other, to live together at all depends on the strength and universality of a useful and commonly shared world view.

Robert H. Allen